MATH-166
Calculus II

Name: ____KEY____

Carefully observe each of the integrals and evaluate them using whatever method you feel is appropriate, you may assume v not is a function of x:

1. $\int (x+2)(x+1)^{1/4} dx$
First let $u = x+1$ and get $du = dx$. Then observing that $x+2 = u+1$ we get the following

$$\int (u+1)u^{1/4} dx = \int u^{5/4} + u^{1/4} dx = \frac{4u^{9/4}}{9} + \frac{4u^{5/4}}{5} + c = \frac{4(x+1)^{9/4}}{9} + \frac{4(x+1)^{5/4}}{5} + c$$

2. $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}} dx$
With this trying $u = \sqrt{x}$ yields $du = 0.5x^{-1/2} dx$. After subbing we get

$$\int 2\sec^2(u) du = 2\int \left(\frac{d}{du}\tan u\right) du = 2\tan u + c = 2\tan\sqrt{x} + c$$

Note that the middle step above uses the Fundamental Theorem of Calculus with the observation that $\frac{d}{dx}\tan x = \sec^2 x$.

3. $\int \sec(x)\tan(x)(\sec(x)-1) dx$
Note that a nice way to simply do this using the Fundamental Theorem of Calculus can be seen by distributing and separating the integral into two parts using the property that integrals are linear. Another way to solve it using u-substitution is by taking $u = \sec(x) - 1$ which gives $du = \sec(x)\tan(x) dx$

$$\int u\ du = \frac{u^2}{2} + c = \frac{(\sec(x)-1)^2}{2} + c$$

4. $\int e^{14x-7} dx$
Knowing that $\frac{d}{dx}e^x = e^x$, a u-substitution of $u = 14x - 7$ makes a lot of sense here. Doing this gives $du = 14dx$ which is the same as $\frac{du}{14} = dx$. Plugging this into the equation gives

$$\int e^u \frac{du}{14} = \frac{1}{14}\int e^u du = \frac{1}{14}\int \left(\frac{d}{du}e^u\right) du = \frac{1}{14}\left(e^u + c\right) = \frac{e^{14x-7}}{14} + c$$

5. $\int_0^{\pi/2} \cos^3(x)\sin(x) dx$
Doing a u-substitution of $\cos x = u$ makes quick work of this problem, since you get
$du = -\sin x\ dx$. This in turn yields the following

$$\begin{aligned}
\int_0^{\pi/2} \cos^3(x)\sin(x) dx &= -\int_{\cos 0}^{\cos(\pi/2)} u^3 du \\
&= -\int_1^0 u^3 du \\
&= \int_0^1 u^3 du \\
&= \left[\frac{u^4}{4}\right|_0^1 \\
&= \frac{1^3}{4} - \frac{0^3}{4} = \frac{1}{4}
\end{aligned}$$

6. $\int \frac{dx}{(1+\sqrt{x})^3}$
While there is no particular reason to try a u-substitution here, attempting one leads to the choice of $u = 1 + \sqrt{x}$. Trying this and seeing where it leads yields the following

$$\begin{aligned}
u &= 1 + \sqrt{x} && \text{Doing prep work for u-substitution} \\
du &= \frac{dx}{2\sqrt{x}} && \\
2\sqrt{x}\,du &= dx && \\
2(u-1)du &= dx && \text{Since there is no } \sqrt{x} \text{ term and } u - 1 = \sqrt{x}
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{(1+\sqrt{x})^3} &= \int \frac{2(u-1)}{u^3} du && \text{Doing the u-substitution} \\
&= \int \frac{2}{u^2} du + \int \frac{-2}{u^3} du && \text{Simplifying} \\
&= \frac{-2}{u} + \frac{1}{u^2} + c && \text{Evaluating the integral} \\
&= \frac{-2}{1+\sqrt{x}} + \frac{1}{(1+\sqrt{x})^2} + c && \text{Plugging in the } x \text{ expression back in for } u
\end{aligned}$$

7. $\int e^{-v^2} dx$
This last one is more of a trick question, I did say in the directions to carefully look at them and that v does not depend on x to further emphasize the difference. Note that the integral doesn't depend on x; thus, we just get $\int e^{-v^2} dx = e^{-v^2} \int dx = xe^{-v^2} + c$. Regarding $f(x) = e^{-x^2}$, it is a very cool type of function called a Gaussian function. Such functions show up frequently in analysis of statistical problems.

If something isn't clear or there is a mistake email me at chase.reuter@ndus.edu